

N70-27453

NASA TECHNICAL TRANSLATION

NASA TT F-12,975

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PLASMA OF FINITE VOLUME IN LINES WITH DOPPLER BROADENING

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Translation of "Pogloshcheniye fotonov i
izlucheniye neravnovesnoy plazmy konechnogo
ob'yema v liniyakh s dopplerovskim ushireniyem".
Izvestiya Vysshikh Uchebnykh Zavedeniy, Fizika,
No. 9, 1969, pp. 54-59.

**CASE FILE
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 MAY 1970

PHOTON ABSORPTION AND RADIATION OF NONEQUILIBRIUM PLASMA OF FINITE VOLUME IN LINES WITH DOPPLER BROADENING

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ABSTRACT. This paper examines the absorption of photons in a plasma of finite volume for the spherical geometry model. The radiation of a plasma confined in a cylindrical volume is obtained in the two-hemisphere approximation.

Let us consider the absorption of photons which are radiated for a given spontaneous transition. Considering the model of a homogeneous plasma confined in a spherical volume of radius R , we take the central point of this volume as the characteristic point and find the absorption at this point. The number of quanta of frequency ν arriving at the point from the entire volume is

$$\int_V P_\nu \frac{\exp(-\kappa_\nu r)}{4\pi r^2} dV, \quad (1)$$

where V is the volume, r is the distance from the point at which the quantum originated to the point in question, P_ν is the emitted radiation spectrum (in number of quanta), κ_ν is the absorption coefficient of photons of the given frequency ν . /54*

In (1) we consider the coefficient κ_ν to be constant throughout the volume. Otherwise, we would have to write $\exp(-\kappa_\nu r)$ in place of $\exp\left(-\int_0^r \kappa_\nu(x, y, z) dr\right)$. At all frequencies of a given broadened line, per unit volume in the region of the given point there is absorbed

$$\Pi = \int_V \int P_\nu \kappa_\nu \frac{\exp(-\kappa_\nu r)}{4\pi r^2} dV d\nu. \quad (2)$$

* Numbers in the margins indicate pagination in the original foreign text.

Writing (2) in the spherical coordinate system and performing the integration over the space variables, we find

$$\Pi = \int_0^{\infty} P_v [1 - \exp(-R\kappa_v)] dv. \quad (3)$$

Let us examine in more detail the frequency characteristics κ_v and P_v which appear here. We have for the absorption coefficient [1, 2]

$$\kappa_v = \kappa_0 \exp \left[-\frac{\mu c^2}{2R^*T} \left(\frac{v - v_0}{v_0} \right)^2 \right], \quad (4)$$

$$\kappa_0 = \left(1 - \frac{g_H}{g_B} \frac{N_B}{N_H} \right) \frac{\pi q^2}{m v_0} f N_H \sqrt{\frac{\mu}{2\pi R^*T}}, \quad (5)$$

where μ is the molecular weight, c the speed of light, R^* the universal gas constant, T the gas temperature, q the electron charge, m the electron mass, f the oscillator force, N the level concentration, and g the statistical weight. /55

The subscript B refers to radiating levels, the subscript H refers to the absorbing levels, 0 refers to the center of the line.

In (5) the factor $\left(1 - \frac{g_H}{g_B} \frac{N_B}{N_H} \right)$ accounts for the induced radiation, which can be considered as an effect which reduces the absorption. We have for P_v [1]

$$P_v = P_0 \exp \left[-\frac{\mu c^2}{2R^*T} \left(\frac{v - v_0}{v_0} \right)^2 \right]. \quad (6)$$

Let us find P_0 (expressed in numbers of quanta). The total number of radiated quanta is $N_B A$, where A is the spontaneous transition probability.

Then

$$P_0 \int_0^{\infty} \exp \left[-\frac{\mu c^2}{2R^*T} \left(\frac{v - v_0}{v_0} \right)^2 \right] dv = N_B A. \quad (7)$$

Let us calculate the integral in (7). Following [1], we make the change of variables

$$\omega = c \sqrt{\frac{\mu}{2R^*T}} \left(\frac{v - v_0}{v_0} \right).$$

Let us consider the limits. For $\nu = +\infty$ $\omega = +\infty$. For $\nu = 0$, we have

$$\omega = -c \sqrt{\frac{\nu}{2R^*T}} \approx -10^6.$$

Considering that this quantity may be equated to $-\infty$, we obtain the integral

$\int_{-\infty}^{\infty} \exp(-\omega^2) d\omega$, but this is the Poisson integral, equal to $\sqrt{\pi}$ (see, for example, [3]). Thus, the integral in (7) is $\frac{\nu_0}{c} \sqrt{\frac{2\pi R^*T}{\nu}}$. However, this derivation is really incorrect, since we have no criterion to be used for equating $c \sqrt{\frac{\nu}{2R^*T}} \approx 10^6$ to infinity, and in this connection the calculated value of the integral cannot be considered reliable. Therefore, we shall examine another derivation. Replacing $\frac{\nu - \nu_0}{\nu_0}$ by x , we find that the integral in (7) equals

$$\int_{-1}^{\infty} \exp\left(-\frac{\nu c^2}{2R^*T} x^2\right) \nu_0 dx,$$

since for $\nu = \infty$ $x = \infty$, and for $\nu = 0$ $x = -1$. This integral may be split into two integrals, writing it in the form

$$\int_0^{\infty} \exp\left(-\frac{\nu c^2}{2R^*T} x^2\right) \nu_0 dx - \int_0^{-1} \exp\left(-\frac{\nu c^2}{2R^*T} x^2\right) \nu_0 dx. \quad (8)$$

Since the integrand in the Poisson integral is symmetrical, the first integral here is $\frac{1}{2} \frac{\nu_0}{c} \sqrt{\frac{2\pi R^*T}{\nu}}$. The second integral is expressed in terms of the probability integral Φ [4]:

$$\begin{aligned} - \int_0^{-1} \exp\left(-\frac{\nu c^2}{2R^*T} x^2\right) \nu_0 dx &= -\frac{\nu_0}{2c} \sqrt{\frac{2\pi R^*T}{\nu}} \Phi\left(-c \sqrt{\frac{\nu}{2R^*T}}\right) = \\ &= \frac{\nu_0}{2c} \sqrt{\frac{2\pi R^*T}{\nu}} \Phi\left(c \sqrt{\frac{\nu}{2R^*T}}\right). \end{aligned}$$

We have already noted that $c \sqrt{\frac{\mu}{2R^*T}} \approx 10^6$.

The probability integral equals one to within 10^{-7} for an argument equal to five, and with increase of the argument this precision increases even more sharply (see, for example, [5]). Thus (8) equals $\frac{\nu_0}{c} \sqrt{\frac{2\pi R^*T}{\mu}}$. Then we find from (7)

$$P_0 = \frac{N_n A}{\frac{\nu_0}{c} \sqrt{\frac{2\pi R^*T}{\mu}}} \quad (9)$$

We note that we shall have need for a comparison of the two methods of calculating the integral in (7) in our later arguments. Let us return to (3). Substituting (4), (6), and (9) into (3), we obtain for the photon absorption

$$\Pi = \frac{N_n A}{\frac{\nu_0}{c} \sqrt{\frac{2\pi R^*T}{\mu}}} \int_0^\infty \exp \left[-\frac{\mu c^2}{2R^*T} \left(\frac{\nu - \nu_0}{\nu_0} \right)^2 \right] \left\{ 1 - \exp \left[-R\kappa_0 \exp \left(-\frac{\mu c^2}{2R^*T} \left(\frac{\nu - \nu_0}{\nu_0} \right)^2 \right) \right] \right\} d\nu, \quad (10)$$

remembering that here κ_0 is defined by (5). We introduce into (10) the variable

$$\omega = c \sqrt{\frac{\mu}{2R^*T}} \left(\frac{\nu - \nu_0}{\nu_0} \right).$$

Then the previously mentioned question of the limits arises, namely: for

$\nu = 0$ $\omega = -c \sqrt{\frac{\mu}{2R^*T}} \approx -10^6$ can we assume approximately that this quantity equals minus infinity? The correctness of this assumption has been proved for the integral in (7), based on the fact that a rigorous analysis leads to the same result as the derivation using this assumption. Let us compare the integrands in (7) and (10) after introducing therein the variable ω . In (7) we have the integral $\int_{-\infty}^\infty \exp(-\omega^2) d\omega$, and while in (10) we have

$$\int \exp(-\omega^2) \{1 - \exp[-R\kappa_0 \exp(-\omega^2)]\} d\omega.$$

(For the moment, we leave the questions of the limit open.) We note that the integrands are symmetric in both cases. The integrand of (10) is that of (7)

multiplied by the difference between unity and the exponent. Since the latter is always negative, the factor in the braces is always less than one and, consequently, the integrand in (10) is always less than that in (7). Therefore, the correctness of the replacement of -10^6 by $-\infty$ in the limit for (7) leads to the correctness of the same replacement in (10). Thus (10) may be transformed to the form

$$\Pi = \frac{N_B A}{V \pi} \int_{-\infty}^{\infty} \exp(-\omega^2) \{1 - \exp[-R \kappa_0 \exp(-\omega^2)]\} d\omega. \quad (11)$$

The following function was introduced and tabulated in [6] (see also [7])

$$f(\xi) = \frac{1}{V \pi} \int_{-\infty}^{\infty} \exp(-\omega^2) \exp[-\xi \exp(-\omega^2)] d\omega. \quad (12)$$

Bearing in mind that the first integral in (11) (after removing the brackets) is the Poisson integral, equal to $\sqrt{\pi}$, we see that if we introduce the function (12), then (11) takes the form

$$\Pi = N_B A [1 - f(\xi)], \quad (13)$$

where

$$\xi = R \kappa_0.$$

Since the absorption coefficient can be expressed in terms of the photon absorption cross section σ_{abs} and the concentration N_H of the absorbing atoms, namely: $k = \sigma_{\text{abs}} N_H$, we have

$$\xi = R \sigma_{\text{abs}} N_H \left(1 - \frac{g_H}{g_B} \frac{N_B}{N_H}\right), \quad (14)$$

and the cross section is

$$\sigma_{\text{abs}} = \frac{q^2 f}{m \nu_0} \sqrt{\frac{\pi \nu}{2 R^* T}}. \quad (15)$$

Since a tabulated function appears in (13), this expression is not always convenient for use in practice, for instance, when (13) must be introduced into the equations, and they must then be solved for the unknowns which appear in

$f(\xi)$. Therefore, we make an approximation of $f(\xi)$. We found that for $\xi \geq 0$ the approximation has the form

$$f(\xi) = \frac{1}{1,940(\xi + 0,548)^{1,095}} \quad (16)$$

to within 10% (with the exception of the interval $0.1 \leq \xi \leq 2.5$, where the accuracy deteriorates to 55%). (If $\xi \geq 1$, the approximation $f(\xi) = 0.50\xi^{-1.10}$ is valid to within 2%; however, the condition $\xi \geq 1$ is not always known a priori and is not always satisfied.)

Thus, the following expression is valid for the number of photon absorption events of the given spontaneous transition per unit volume per unit time near the central point of the homogeneous spherical plasma volume

$$\Pi = N_b A \left\{ 1 - 0,516 \left[R \sigma_{0\text{abs}} N_H \left(1 - \frac{g_H N_b}{g_b N_H} \right) + 0,548 \right]^{-1,095} \right\}, \quad (17)$$

where $\sigma_{0\text{abs}}$ may be found from (15).

Let us consider the line radiation of a homogeneous plasma confined in a /58 cylindrical volume of radius R and length l . We use the Biberman two-hemisphere approximation [8]. In this approximation, it is considered that at a point located at the distance x from the center of an infinitely long volume of radius R the radiation comes from two hemispheres — one with radius $R + x$, the other with radius $R - x$. If photons from the hemisphere arrive at the point, then in accordance with (17) the following number of photons is absorbed per unit volume per unit time

$$C_1 = \left[1 - \frac{1}{1,940(C_2 r + 0,548)^{1,095}} \right], \quad (18)$$

where

$$C_1 = \frac{1}{2} N_b A_{b-H}, \quad C_2 = \sigma_{0\text{abs}} N_H \left(1 - \frac{g_H N_b}{g_b N_H} \right),$$

and r is the radius of the given hemisphere.

Then the following number of photons is absorbed in an elementary annular cylinder of length l per unit time

$$2\pi l x dx C_1 \left\{ 2 - \frac{1}{1,940 [C_2 (R - x) + 0,548]^{1,095}} - \frac{1}{1,940 [C_2 (R + x) + 0,548]^{1,095}} \right\}.$$

In order to find the absorption throughout the entire volume, we must integrate this expression in the limits from 0 to R . The total radiation from the entire volume without absorption is

$$N_B A_{B-H} \pi R^2 l = 2\pi l C_1 R^2.$$

In order to obtain the plasma radiation energy Q , we must subtract from the photon radiation the photon absorption, multiply by their energy $h\nu$ (h is the Planck constant), and first sum over all the lower levels which combine with the given upper level and then sum the resulting expression over all the upper levels.

As a result, we obtain

$$Q = 1,03\pi l h \sum_B \sum_H \frac{C_1}{C_2^2} \nu_{B-H} \times \left\{ \frac{(2C_2 R + 0,548)^{0,905} - 2(C_2 R + 0,548)^{0,905} + 0,580}{0,905} + \right. \\ \left. + \frac{C_2 R + 0,548}{0,095} [(2C_2 R + 0,548)^{-0,095} - 2(C_2 R + 0,548)^{-0,095} + 1,060] \right\}. \quad (19)$$

Usually it is sufficient in practice to consider only a few lower transitions. This assumption is satisfied for an infinitely long cylinder, and it is reasonable for use in those cases when $2R < 1$. In the case when $2R > 1$, we can draw an analogous conclusion in the two-hemisphere approximation for an infinite volume confined between two planes located at the distance l from one another. In this case, the integration is performed with respect to the l coordinate.

As a result of the derivation, we obtain

$$Q = 10,9\pi R^2 h \sum_b \sum_n \frac{C_1}{C_2} v_{b-n} [1,060 - (C_2 l + 0,548)^{-0,095}], \quad (20)$$

where the quantities C_1 and C_2 are expressed just as in the preceding derivation. 59
For comparable values of $2R$ and l , it is reasonable to average these two expressions as follows

$$Q = \frac{2R}{2R+l} Q_{2R>l} + \frac{l}{2R+l} Q_{2R<l}.$$

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Received 17 December 1968